Late News of Sylver Coinage

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> "What?" cried Keawe, "two cents? Why, then, you can only sell it for one. And he who buys it—"
>
> —Stevenson, "The Bottle Imp"

1 Introduction

Here I report recent progress in computing Sylver Coinage positions. For definitions, previous results, and bibliographic references, see my 1991 paper [1] (available by request).

2 Dead or Alive

Of the ten "wanted" positions, the eight with $g = 2$ have all been brought to justice. They appear (among others) in Table 1, except for $\{10, 32, 44, 48\}$ [189], which is not "long" enough to be included. Most of the very long positions were solved with the algorithm used in the proof of the Pattern Theorem. My old method of bitmaps and brute force would have taken years to solve $\{8, 30, 34\}$.

The positions with $g = 3$ are still at large. While $\{12, 15, 21\}$ has occasional near-misses, $\{9, 15, 21\}$ has been computed into the thousands with no hint of success. Meanwhile I have promoted $\{10, 32, 66\}$ and $\{12, 26, 70\}$ to the wanted list.

3 Four

For the record, Table 2 shows how the first few even 4-positions behave. Here

$\{6,32,58\}$	$[4,17767]$	$\{10,26,44\}$	$[2383]$
$\{6,34,62\}$	$[4,58905]$	$\{10,26,58\}$	$[42,1399]$
$\{6,40,68\}$	$[4,18749]$	$\{10,26,68\}$	$[3899]$
$\{6,44,76\}$	$[4,2879]$	$\{10,32,48,54\}$	$[4539]$
$\{8,26,30\}$	$[12,4363]$	$\{12,26,58\}$	$[2397]$
$\{8,30,34\}$	$[49337]$	$\{14,22,26\}$	$[931]$
$\{10,18,44\}$	$[12,1881]$	$\{14,26,30,32,36,38\}$	$[999]$
$\{10,26,42,48\}$	$[4455]$	$\{14,26,32\}$	$[1513]$

Table 1: Solutions to some very long positions.

	period	*start*		*period*	*start*
$\{4,6\}$	4	5	$\{4,34\}$	72	101
$\{4,10\}$	8	5	$\{4,38\}$	72	179
$\{4,14\}$	8	15	$\{4,42\}$	40	183
$\{4,18\}$	8	13	$\{4,46\}$	40	249
$\{4,22\}$	24	27	$\{4,50\}$	48	249
$\{4,26\}$	24	23	$\{4,54\}$	336	245
$\{4,30\}$	72	55	$\{4,58\}$	336	323

Table 2: Surface behavior of 4-positions.

are some more Sprague-Grundy numbers for 4-positions:

n	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59
$\mathcal{G}(\{4,n\})$	32	34	20	37	41	42	46	47	51	53	54	49	58	61	63

n	22	26	30	34	38	42	46	50	54	58	62	66	70
$\mathcal{G}(\{4,n\})$	7	10	11	12	13	15	17	18	19	20	22	24	26

Note the dips in the first table and the occurrence of 20 in both tables. No pattern is emerging; the value of $\mathcal{G}(\{4\})$ may still be $\omega+1$.

4 Six

The lowly move 6 appears in four of the very long positions. Table 3 shows the odd winning moves for some 6-positions with $g=2$.

Remarkably, $\{6,40,74\}$ has no odd winning move; at 381091 it enters a period of length 403200. This, the first new long position with $g=2$ and no odd winning move in many years, shows that the Pattern Theorem cannot be safely ignored.

5 Eight, Ten

We now know the value of every 8-position with $g > 1$! This leaves no more 8-positions of interest.

We also know all the winning moves in $\{10\}$. The second-order analysis is complete except for $\{10, 32, 66\}$.

6 The First Two Moves

Further computation has expanded the table of $\{m, n\}$ positions; see Tables 4 and 5. The symbols !? and ?? mean probably winning and losing; in Table 5, @ denotes a degenerate position. Note the new $\mathcal{P}$-position $\{10, 26\}$.

7 Out Of Gas

So far as I know, I am the only person doing Sylver Coinage research, and I am about to abandon it because Bell Laboratories will no longer support it. If you are interested in Sylver Coinage and can spend weeks of background computing time on it, you are welcome to copies of my programs.

References

[1] G. L. Sicherman, "New Results in Sylver Coinage," privately distributed, December 7, 1991.

$\{6,8,10\}$	$[7,11]$	$\{6,26,46\}$	$[11]$	$\{6,52,62\}$	$[51,55]$
$\{6,10,14\}$	$[17,25]$	$\{6,32,46\}$	$[33,43]$	$\{6,58,62\}$	$[9]$
$\{6,14,16\}$	$[7,15]$	$\{6,38,46\}$	$[17,21]$	$\{6,38,64\}$	$[17]$
$\{6,16,20\}$	$[7,9,17]$	$\{6,44,46\}$	$[15,37]$	$\{6,44,64\}$	$[39]$
$\{6,14,22\}$	$[23,27]$	$\{6,28,50\}$	$[13]$	$\{6,50,64\}$	$[33,41]$
$\{6,20,22\}$	$[13,15]$	$\{6,34,50\}$	$[77,81,85]$	$\{6,56,64\}$	$[81,95,109]$
$\{6,16,26\}$	$[7]$	$\{6,40,50\}$	$[55,57,59]$	$\{6,62,64\}$	$[15,37,71]$
$\{6,22,26\}$	$[9,11,25]$	$\{6,46,50\}$	$[9,25,29]$	$\{6,40,68\}$	$[18749]$
$\{6,20,28\}$	$[47,63]$	$\{6,32,52\}$	$[83]$	$\{6,46,68\}$	$[25]$
$\{6,26,28\}$	$[11,13,15]$	$\{6,38,52\}$	$[17]$	$\{6,52,68\}$	$[63,85,113]$
$\{6,22,32\}$	$[37,41]$	$\{6,44,52\}$	$[33,35]$	$\{6,58,68\}$	$[29,33,49]$
$\{6,28,32\}$	$[9,13,17]$	$\{6,50,52\}$	$[31,39,71]$	$\{6,64,68\}$	$[9,29]$
$\{6,20,34\}$	$[39]$	$\{6,34,56\}$	$[237]$	$\{6,38,70\}$	$[17]$
$\{6,26,34\}$	$[11,27,31]$	$\{6,40,56\}$	$[69,85]$	$\{6,44,70\}$	$[155]$
$\{6,32,34\}$	$[15]$	$\{6,46,56\}$	$[25,41,57]$	$\{6,50,70\}$	$[111,119,133]$
$\{6,22,38\}$	$[469]$	$\{6,52,56\}$	$[9,29,37]$	$\{6,56,70\}$	$[63,101]$
$\{6,28,38\}$	$[13]$	$\{6,32,58\}$	$[17767]$	$\{6,62,70\}$	$[33,35]$
$\{6,34,38\}$	$[9,17,25]$	$\{6,38,58\}$	$[17]$	$\{6,68,70\}$	$[21,29]$
$\{6,26,40\}$	$[11]$	$\{6,44,58\}$	$[45,97]$	$\{6,40,74\}$	$[\,]$
$\{6,32,40\}$	$[21,23]$	$\{6,50,58\}$	$[27,43]$	$\{6,46,74\}$	$[25]$
$\{6,38,40\}$	$[17,33]$	$\{6,56,58\}$	$[15]$	$\{6,52,74\}$	$[129,143]$
$\{6,28,44\}$	$[13]$	$\{6,34,62\}$	$[58905]$	$\{6,58,74\}$	$[57,71]$
$\{6,34,44\}$	$[47]$	$\{6,40,62\}$	$[233]$	$\{6,64,74\}$	$[27,55]$
$\{6,40,44\}$	$[9,25,29]$	$\{6,46,62\}$	$[25,45,65]$	$\{6,70,74\}$	$[9,37,53]$

Table 3: Odd winning moves in 6-positions.

2	$[3]$	9	$[6]$
3	$[2]$	10	$[5,14,26]$
4	$[6]$	11	$\mathcal{P}$
5	$\mathcal{P}$	12	$[8,$
6	$[4,9]$	13	$\mathcal{P}$
7	$\mathcal{P}$	14	$[7,8,10,$
8	$[12,14]$	15	$[5,$

Table 4: Solutions to positions of the form $\{m\}$.

	6	7	8	9	10
20	[4]	[92]	[12,26,30]	[61,95]	@
21	[9]	@	[9]	[6,8]	[128]
22	[4]	[61]	[10,14]	[14,17]	[8]
23	[13]	[55]	[11,17]	[74,139]	[48,54]
24	@	[9,11,26]	@	[6,7]	[11,14,16]
25	[46]	[47,83]	[13]	[32]	[5]
26	[4,11]	[24,58]	[20]	[128]	$\mathcal{P}$
27	[9]	[127]	[33,55]	@	[62]
28	[4,13]	@	[12,14,18]	[67,87]	[11,13,14]
29	[68]	[153]	[30]	[86,102]	[64,66]
30	@	[94]	[14,20,29]	[6,11]	@
31	[47]	[158]	[185]	[5]	[54,119]
32	[4]	[122]	@	[25,101]	[48,66??]
33	[9]	[149]	[27]	[6]	[128,217]
34	[4]	[11]	[35,36]	[245]	[14]
35	[62]	@	[34]	[164]	[5]
36	@	[39,76]	[12,14,34]	@	[26,

	11	12	13	14	15
12	[49]				
13	[9,19]	[59]			
14	[93]	[16]	[155]		
15	[102]	[18,21??]	[17,55]	[62,66]	
16	[57,58]	[8,14,18,22]	[137,140]	[8,12]	[7]
17	[24,71]	[50,105]	[15,27]	[38,74]	[13,29]
18	[68,118]	[10,15,16,21]	[164]	[5]	[12]
19	[13]	[61]	[11]	[59]	[198]
20	[46,63]	[8,34,38,42]	[35,56]	[10]	[5]
21	[94,167]	[15??,18,27??]	[239]	[7]	[12??,
22	@	[16,30]	[53,59]	[8,9]	[
23	[8]	[135,146]	[6]	[	[
24	[7,10,17]	@	[	[8,10,	[
25	[81,123]	[177]	[8,	[27,32,	[5]
26	[6]	[28,70??]	@	[	[
27	[79]	[21??,42!?]	[17,	[25,	[
28	[10,46]	[8,26,	[6,10]	@	[

Table 5: Solutions to positions of the form $\{m, n\}$.